

Separable	$f(x) = g(y) \frac{dy}{dx}$	Separate $\frac{dy}{dx}$ and then integrate
Linear	$y' + a(x)y = b(x)$	Multiply by $e^{A(x)}$ where $A(x) = \int a(x)dx$
Exact	$M(x, y) + N(x, y) \cdot y' = 0$	Test: $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Find $f$ where $\frac{\partial f}{\partial x} = M$ and $\frac{\partial f}{\partial y} = N$ Solution: $f(x, y) = c$
Constant coefficient	$a_2y'' + a_1y' + a_0y = 0$	Two real roots $r_1, r_2$ use $e^{r_1x}$ and $e^{r_2x}$ If double real root $r$ use $e^{rx}$ and $xe^{rx}$ If complex roots $r = \alpha \pm \beta i$ use $e^{\alpha x} \cos(\beta x)$ and $e^{\alpha x} \sin(\beta x)$
Variation of parameters	$y'' + a_1(x)y' + a_0(x)y = b(x)$  $y_1$ and $y_2$ sols. to homogeneous eqn.	$y_p = v_1y_1 + v_2y_2$  $v_1 = \int \frac{-y_2 \cdot b(x)}{W(y_1, y_2)} dx$ $v_2 = \int \frac{y_1 \cdot b(x)}{W(y_1, y_2)} dx$
Reduction of Order	$y'' + a_1(x)y' + a_0(x)y = 0$  on the interval $I$	$y_1$ is a solution that isn't zero on $I$  $y_2 = y_1 \cdot \int \frac{e^{-\int a_1(x)dx}}{y_1^2} dx$
Euler's method	$y' = f(x, y)$ $y(x_0) = y_0$	$x_n = x_{n-1} + h$ $y_n = y_{n-1} + h \cdot f(x_{n-1}, y_{n-1})$

$b(x)$	$y_p$ guess for undetermined coefficients
constant	$A$
$5x - 3$	$Ax + B$
$10x^2 - x + 1$	$Ax^2 + Bx + C$
$\sin(6x)$	$A \cos(6x) + B \sin(6x)$
$\cos(6x)$	$A \cos(6x) + B \sin(6x)$
$e^{3x}$	$Ae^{3x}$
$(2x + 1)e^{3x}$	$(Ax + B)e^{3x}$
$x^2e^{3x}$	$(Ax^2 + Bx + C)e^{3x}$
$e^{3x} \sin(4x)$	$Ae^{3x} \cos(4x) + Be^{3x} \sin(4x)$
$e^{3x} \cos(4x)$	$Ae^{3x} \cos(4x) + Be^{3x} \sin(4x)$

Taylor series:  $f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f^{(3)}(x_0)}{3!}(x - x_0)^3 + \frac{f^{(4)}(x_0)}{4!}(x - x_0)^4 + \dots$

$$\int u dv = uv - \int v du \quad \int \sin(x)dx = -\cos(x) \quad \int \cos(x)dx = \sin(x) \quad W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1'$$